

MATHEMATICS: CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

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1. Given $f(x) = x^3 - 6x^2 + 9x$ and $g(x) = 4$.
- Find the coordinates of the points common to the graphs of f and g .
 - Find all the zeros of f .
 - If the domain of f is limited to the closed interval $[0, 2]$, what is the range of f ? Show your reasoning.
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2. A particle moves on the X -axis so that its acceleration at any time $t > 0$ is given by $a = \frac{t}{8} - \frac{1}{t^2}$.
- When $t = 1$, $v = \frac{9}{16}$, and $s = \frac{25}{48}$.
- Find the velocity v in terms of t .
 - Does the numerical value of the velocity ever exceed 50? Explain.
 - Find the distance s from the origin at time $t = 2$.
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3. Given the curve $x + xy + 2y^2 = 6$.
- Find an expression for the slope of the curve at any point (x, y) on the curve.
 - Write an equation for the line tangent to the curve at the point $(2, 1)$.
 - Find the coordinates of all other points on this curve with slope equal to the slope at $(2, 1)$.
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4. (a) What is the set of all values of b for which the graphs of $y = 2x + b$ and $y^2 = 4x$ intersect in two *distinct* points?
- (b) In the case $b = -4$, find the area of the region enclosed by $y = 2x - 4$ and $y^2 = 4x$.
- (c) In the case $b = 0$, find the volume of the solid generated by revolving about the X -axis the region bounded by $y = 2x$ and $y^2 = 4x$.
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5. (a) Find the coordinates of the absolute maximum point for the curve $y = xe^{-kx}$ where k is a fixed positive number. Justify your answer.
- (b) Write an equation for the set of absolute maximum points for the curves $y = xe^{-kx}$ as k varies through positive values.
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6. A manufacturer finds it costs him $x^2 + 5x + 7$ dollars to produce x tons of an item. At production levels above 3 tons, he must hire additional workers, and his costs increase by $3(x - 3)$ dollars on his total production. If the price he receives is \$13 per ton regardless of how much he manufactures and if he has a plant capacity of 10 tons, what level of output maximizes profits?